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A SEARCH FOR LOW AV EARTH-TO-MOON TRAJECTORIES

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Abstract

A search for low AV 1 Earth-to-Moon trajectories has been initiated. Numerical integration of the equations of motion from the circular restricted three-body problem has resulted in the computation of a trajectory that saves more than 100 m/s over a Hohmann transfer, although the flight time is almost ten months. The approach used involves the computation of two trajectory "legs": first, a trajectory from low 1 Earth orbit to the L_1 libration point of the Earth - Moon system, and second, a trajectory from L_1 to orbit about the Mom. Multiple orbits about Earth using lunar perturbations facilitates the transfer to L_1. Similarly, the L_1 to Moon leg uscs the perturbation from the 1 Earth to achieve a low orbit about the Moon. Small maneuvers arc used in both legs 10 control the orbital period so the third body perturbations can be used advantageously.

Introduction

Most spacecraft on 1 Earth-to-Moon trajectories arc limited in their mass by the propulsive requirements. Of course, any reductions in the propulsive requirements arc beneficial since more mass (e.g., scientific instruments or humans) can be delivered m the final destination. This problem has been studied extensively since, and even before, space travel began. Such research has produced a number of useful trajectory designs and trajectory analysis tools. In recent work by Sweetser[1] the minimum total AV required to reach the Moon from Earth (as a lower bound) was quantified. "Ibis research, however, did not Produce the actual trajectory that uses the. minimum AV amount, assuming that one even exits. Nevertheless, the results (applicable to the circular restricted three-body problem) do reveal some conditions on the trajectory that would use the minimum AV. The current effort, then, give.n the minimum AV amount, is to find a trajectory that uses that minimum (or at least is "close" m it). If the. time-of-flight is prohibitively long for practical applications, a transfer trajectory can be sought that is near the minimum but that has a shorter travel time. This study has focused on trajectories computed using some of the conditions given in Reference 1 that arc required by the minimum AV trajectory. These conditions do not completely specify the number of maneuvers, their locations, magnitudes, and directions. Thus, determining the strategy to define each maneuver is one of the more important aspects of this work. While a trajectory has not yet been found that uses the minimum AV, some relatively low AV cases have been obtained and arc the subject of this paper.

Previous Contributions

1 Early efforts on the 1 Earth-to-Moon transfer problem include works by 1 Egorov2 Bucheim[3], and NASA in the Lunar Flight Handbook[4]. These approaches quantify the minimum velocity required near the Earth to achieve a transfer to the Moos within the context of the restricted three-body problem. In addition, direct two-impulse

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trajc'dories have been computed by many using patched-conic and/or numeric.al integration methods that do not minimize AV but do admit free-rcham trajectories and arc therefore more useful for human-based missions.

A new class of translunar trajectories has recently been found by Belbruno and Miller[5-7] that uses the solar perturbation to lower total AV, These trajectories have flight times of three to four months and save. approximately 100 m/s over dire.c[ballistic transfers.

la seeking fuel-cffic.ient trajectories to the Moon, Sweetser first identified the. minimum total required AV to leave an orbit about the 1 :arth and insert into lunar mbit.1 Using the restricted three-body problem and Jacobi's constant, a lower bound on required AV was determined. (The actual figure depends on the initial 1 :arth orbit, the final lunar orbit, and their inclinations.)

Model: The Restricted Three-Body Problem

The equations governing motion in this problem arc written in the form associated with the circular restricted three-hdy problem. In defining the usual rotating coordinate system, the x-axis is directed from the barycenter to the smaller primary (i.e., barycenter to Mom). The y-axis is rotated 90° from the x-axis in the primary plane of motion, The z-axis completes the right-handed frame, defining the out-of-plane direction. The problem is nondimensionalized in the usual manner in which the distance between the primaries, the sum of their masses, the angular velocity of the rotating frame, and the universal gravitational constant arc all unity. The parameter μ is defined as the ratio of the smaller primary mass to the sum of both primaries' masses.

Let the vector p describe the position of [he. spacecraft (assumed to be an infinitesimal mass) from the barycenter such that p has components x, y, and z. The equations of motion, assuming circular primary motion, can be written

$$\ddot{x} - 2\dot{y} = \frac{\partial U}{\partial x}$$

$$\ddot{y} + 2\dot{x} = \frac{\partial U}{\partial y} \qquad (1)$$

$$\ddot{z} = \frac{\partial U}{\partial z}$$

where

$$U = (x^2 + y^2)/2 + (1-\mu)/d \quad p / r$$

$$d = \left[(x + \mu)^2 + y^2 + z^2\right]^{1/2}$$

$$r = \left[(x - 1 + \mu)^2 + y^2 + z^2\right]^{1/2}.$$

These equations arc well knows and their derivation is widely available.[8]

Two other aspects of the restricted three-bml y problem arc of particular importance to this work. An integral of motion was given by Jacobi as

$$C = 2U - (\dot{x}^2 + \dot{y}^2 + \dot{z}^2), \qquad (2)$$

and is used in quantifying the minimum AV required to transfer from Earth orbit to lunar orbit. Also, five equilibrium points or libration points were found by Lagrange in 1772. The 1,1 or interior libration point located on the Earth-Moon line between the Earth and Moon (58,071.6 km from the Moon) is specifically used here.

Search Strategy

The search for low AV Earth-to-Moon trajectories has been attempted primarily through numerical means, i.e., propagation by numerical integration of the equations of motion. Transfers have been sought that originate in a low Earth orbit of 167 km altitude and terminate in a lunar circular orbit of 100 km altitude. In this study, all motion has been contained in the plane of the motion of the primaries. The software used to analyze this problem has been structured to allow convenient trial-and-error inputs of AVS at specific locations,

Preliminary Considerations

The work by Sweetser reveals certain conditions on the minimum AV 1 :arth-to-Moon transfer. First, the trajectory must pass through the I.] point, and, at the L_1 point the velocity relat ive to the mating frame must be zero. Thus as the spacecraft approaches 1.1 from the Earth, it apparently "slows down", asymptotically approaching zero velocity relative to the rotating frame. The spacecraft then proceeds to the Moon by departing the 1,1 point asymptotically from zero velocity. Of course, the asymptotic arrival/departure at L_1 would result in an infinite time-of-flight,

Another condition states that the change in Jacobi's constant as a result of a maneuver is maximized if the AV is performed where the velocity relative to the rotating frame is greatest and is performed in the same dire.clioa as the velocity relative to the rotating frame. As shown in 1, maximizing the change in Jacobi's constant as the result of a maneuver is generally desirable in transferring from low Earth orbit to L_1 and from L1 to lunar orbit.

The Goal

The minimum ΔV to travel from low Earth orbit to lunar orbit, assuming motion governed by the circular restricted three-body problem, is given by Sweetser[1] as

$$\Delta V_{min} = \Delta V_E + \Delta V_M, \qquad (3)$$

where

$$\Delta V_E = [\delta C_E + V_E^2]^{1/2} - V_E$$

and

$$\Delta V_M = [\delta C_M + V_M^2]^{1/2} - V_M.$$

The quantity ΔV_E represents the minimum ΔV to transfer from low Earth orbit to L_1 and ΔV_M is the minimum to transfer from L_1 to lunar orbit (where the trajectory arrives at L_1 from Earth with zero velocity relative to the rotating frame before proceeding to the Moon, as described in the previous paragraph). δC_E is the change in Jacobi's constant from low Earth orbit to L_1 and V_E is the velocity in the low Earth orbit relative to the rotating frame. Similar definitions apply for δC_M and V_M. Using the relationships given in (3) for the case mentioned previously, transferring from a circular low Earth orbit of 167 km altitude to L1 requires at least 3.099 km/s and from L1 to a 100 km altitude circular lunar orbit of zero inclination requires at least 0.627 km/s.

The Strategy

Given the conditions for a minimum ΔV transfer, trajectories have been sought originating from low Earth orbit that pass through the Earth-Moon L_1 libration point with almost no velocity (relative to the rotating frame) and that enter into orbit about the Moon. (As mentioned previously, trajectories that arrive at L1 with no velocity relative to the rotating frame would have infinite flight times, thus, in an effort to obtain practical flight times, small arrival velocities at L1 were used.) Each trajectory is "formed" by computing two "legs": first, from low Earth orbit (circular orbit of 167 km altitude) to the L1 point, and second, from the L_1 point to lunar orbit (circular of 100 km altitude). Discrete ΔVs are applied at perigee/perilune locations to control the orbit periods so that the perturbing gravitational force from the third body (Moon or Earth) can be used advantageously. Although not specifically computed *here*, trajectories from the Moon to the Earth could be found using a strategy similar to that described in the following paragraphs.

Earth-to-L_1 Leg

Computing a maneuver or series of maneuvers for a trajectory originating in low Earth orbit to arrive at the L1 point with a small velocity magnitude would require some type of targeting algorithm. To avoid this complication, Earth-to-L_1 legs have been found by numerically integrating backward in time starting at L_1 with a small velocity (~1 m/s) that is just enough to "push" the trajectory toward Earth (instead of the Moon). The trajectory then "settles" into a large near-elliptic orbit about the Earth (semimajor axis ~205,000 km) that is perturbed by the Moon. The strategy used here is to implement ΔVs (oriented in the direction of the velocity with respect to the rotating frame) at certain perigee locations that adjust the orbit period to control the lunar perturbation effects.

The initial elliptic orbit (going backward in time) has a period of approximately 10.7 days. This results in a 5:2 resonant behavior in which the Moon completes two orbits while the spacecraft trajectory completes five. Thus in five orbits the orbit spacecraft apogee will again be oriented approximately toward the Moon along the x-axis (and will also pass relatively close to the Moon). The orientation of the line of apsides to the x-axis is critical in determining how the lunar perturbation changes subsequent orbits. To quantify these effects, the angle α (-180° < α < 180°) is defined that represents the angle between the Earth-Moon line and the Earth-spacecraft line when the spacecraft is at apogee. α is defined as zero when the spacecraft apogee line lies on the Earth-Moon line and is directed toward the Moon. Positive α is defined such that the Moon "trails" the spacecraft as the spacecraft reaches apogee (for a "time forward" perspective).

Experimentation by implementing small maneuvers (~± 5 m/s) at the first (as integrating backward) perigee point revealed some general trends on the lunar perturbation effects as related to α. As integrating backward, a value of α between 0 and +30 degrees produces a subsequent significant perigee increase and apogee reduction on the next orbit revolution. If α is between 0 and -30 degrees, the next perigee and apogee decrease significantly. In addition, the magnitude of the reductions and increases are much greater at apogees close to L_1, and, in general, the perigee changes are greater than the apogee changes. If, as a result of a maneuver, apogee becomes too high, the subsequent path may leave the Earth's vicinity and enter into orbit about the Moon. Also, as α gets larger the changes in perigee and apogee are not as great since the spacecraft dots not pass as dose. to the M (inn.

The strategy used is a balancing act to try to maximize perigee drops while avoiding being drawn into the Moon's vicinity, By using small maneuvers at perigee locations, the orbit periods can be changed slightly which acts to control the value of α. In particular, when the spacecraft apogee is oriented toward the Moon (a small), a maneuver is used (when necessary) at some previous perigee to adjust α to lie in the approximate range of $-10° < \alpha < 0°$. If *a* is too large (e.g., 5° or 100), a negative maneuver (energy and period reducing) is used. When necessary, positive maneuvers arc used even though they add energy to the orbit, since this undesirable effect (as proceeding backward in time) is far exceeded by the benefits of the enhanced lunar perturbation effect of perigee drop.

As perigee drops, the semimajor axis decreases and thus the orbit period dcx-masts as well. When this occurs a 3:1 commensurability between the lunar and spacecraft orbits is established and is maintained until the perigee is reduced to the required altitude for injection into the circular low Earth orbit. A large AV is used for this final maneuver (which, of course, is actually the first maneuver out of low Earth orbit with time progressing forward).

L_1-to-Moon Leg

The L_1-to-Moon leg is computed using a strategy similar to the Earth-to-L_1 leg. in this case, the numerical integration proceeds forward in time. A small initial velocity is specified at the libration point which is just enough to "push" the spacecraft into an orbit about the Moon (instead of the Earth). Thus when the two trajectory legs arc "patched" together a small velocity discontinuity will exist at L_1, requiring a small AV at this point.

The Earth now acts as the third body and is used to reduce perilune to the lunar orbit distance. A relatively large maneuver is performed at perilune to circularize the orbit about the Moon.

Results

Constants

The following constants were used in this study:

a = 384,748 km (mean semimajor axis of lunar orbit)

a = 2.661700x10-6 rads/sec (mean motion of Moon)

$\mu_M = 4{,}902.79\ km^3/s^2$ ((gravitational constant limes lunar mass)

$\mu_E = 398{,}600.49\ km^3/s^2$ (Gravitational constant times Earth mass).

These value.s produce μ = 1.2150557 x10-2 and an L_1 location 58.071.6 km from the Mom.

Earth-to-L_1 Leg

The lowest AV case obtained m date requires a total AV amount of 3.1937 km/s to travel from low Earth orbit to L1. Six maneuvers arc used over a time-of-flight of 239.76 days. The spacecraft arrives at L_1 with a velocity of 0.95 m/s in the negative y direction. This trajectory leg exceeds the theoretical minimum of 3.099 km/s by 95 rids) leaving the challenge to further reduce the AV while also reducing the lengthy lime of flight,

L_1-to-Moon Leg

In this leg, the AV was much more easily reduced, almost to the theoretical minimum value. of 627 rids. This success may be duc to the relatively larger perturbing effect of Earth's gravity. The lowest AV case uses three maneuvers and totals 629.4 m/s over S2.26 days. The spacecraft departs L1 with a velocity of 0.1 m/s in the x direction. In pursuit of a more useful trajectory, future efforts could try to reduce the. flight time while attempting to retain the near-minimum AV amount.

The Patched Trajectory

Figure 1 shows both legs of the trajectory relative to the rotating frame. The origin of the plot corresponds to the barycenter of the Earth-Moon system, and the x and y axes arc as defined previously for the rotating frame. The trajectory begins with the first maneuver used to depart the low Earth orbit ("Earth injection"), and ends with the maneuver used to insert into the circular orbit about the Moon ("Moon insertion"). Figure 2 shows the same trajectory in the inertial frame, with the. origin at the barycenter as in Figure 1. In Figure 2, then, the x and y axes represent inertially fixed directions. Table 1 displays a time-history of the trajectory starting from the injection maneuver at time zero out of low Earth orbit to insertion into the circular lunar orbit. The total AV, including the small maneuver of 0.96 m/s at the L1 point, is 3.8240 km/sec. The total transfer time is 292.02 days,

Figure 3 shows an expanded view of the L_1-to-Moon leg in the rotating frame. In this figure the origin has been moved m the center of the Mom.

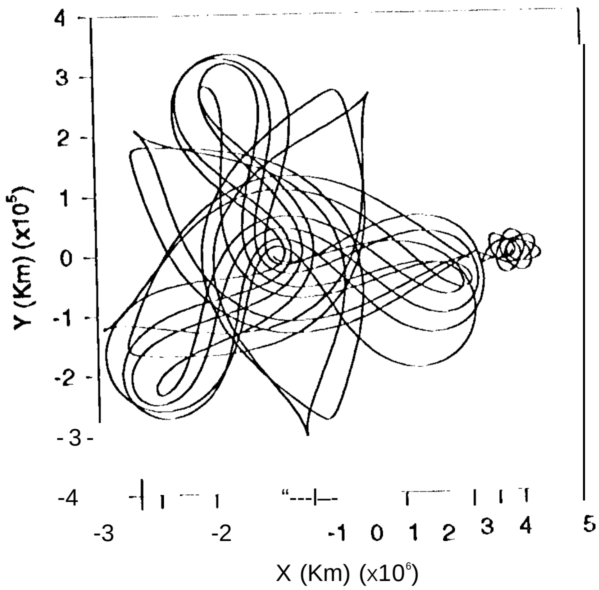

Figure 1. Earth-to-Moon Trajectory Shown in the Rotating Frame

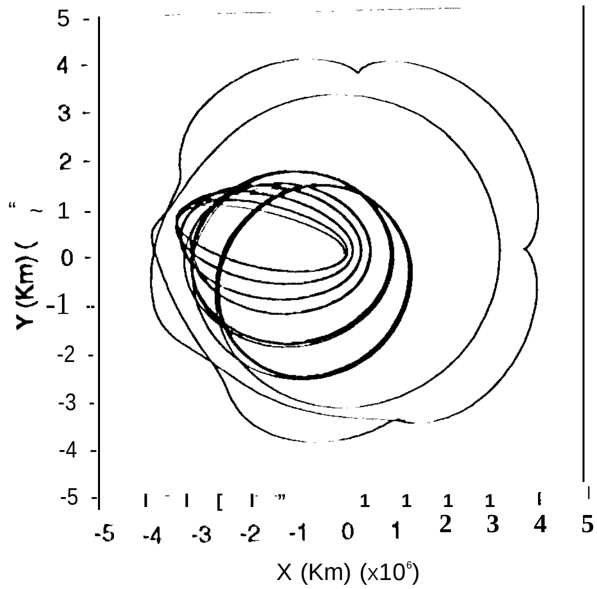

Figure 2. Earth-to-Moon Trajectory Shown in the Inertial Frame

Table 1. Time History Of Earth-to-Moon Trajectory.

Time (Days)	ΔV (m/s)	ΔV Location	Distance to Perigee/Perilune (km)
0	+3126.008	LEO	6,545
27	+8.673	Perigee #1	12,983
54	+16.000	Perigee #3	24,379
82	+25.000	Perigee #7	42,020
162	+16.000	Perigee #15	104,651
219	-2.000	Perigee #20	126,509
240	+0.955	[illegible]	N/A
255	+1.000	Perilune #1	[illegible]
270	+1.215	Perilune #4	7,594
292	-627.181	Lunar Orbit	1,838
TOTAL ΔV	3.824 km/sec		

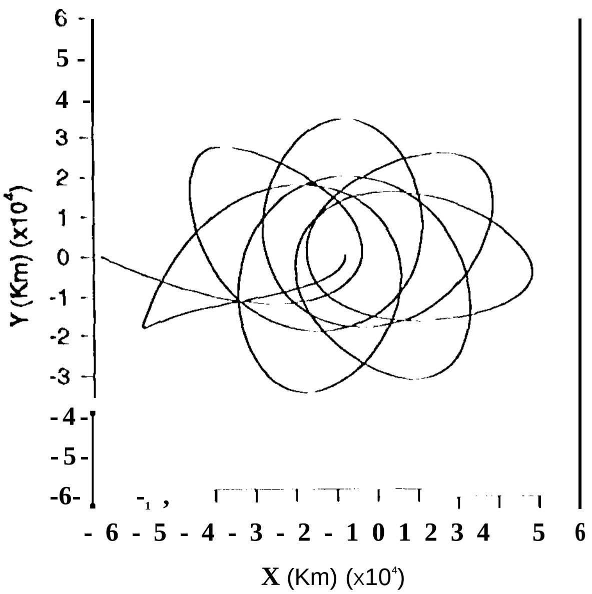

Figure 3. L_1-to-Moon Leg Shown in the Rotating Frame

A Trajectory with Lower Total ΔV

The Earth-to-L_1 leg of the trajectory described above uses a 3:1 commensurability between the spacecraft orbit about Earth and the lunar orbit. By using a 4:1 commensurability the ΔV from the Earth to the, Moon was reduced by approximately 13 m/s to a total of 3.81122 km/s. However, the reduction in ΔV come.s with an increase in total time-of-flight 10506.61 days, making this trajectory an unlikely choice for an actual mission design.

Discussion

Different aspects need to be considered when assessing the merits and deficiencies of transfer trajectories from the Earth to the Moon. Of course, when humans are being transported the low ΔV trajectory found in this study would not be suitable due to the lengthy time-of-flight. However, in missions where short flight times are not as critical, this type of trajectory may be of use. Examples include "cargo" missions delivering a large payload mass to the Moon and spacecraft requiring as much payload capacity as possible. in both of these cases the long flight times may be palatable to mission designers.

Table 2 shows a summary of Earth-to-Moon transfer trajectories. The total ΔV, time-of-flight, Earth injection ΔV, and lunar insertion ΔV are listed. It should be noted that unlike the final equatorial orbit about the Moon used here, the Belbruno-Miller trajectory terminates in a circular *polar* orbit.

The trajectory computed in this study features a lunar orbit insertion ΔV that is 192 m/s less than that for a Hohmann transfer, potentially important for maximizing payload capacity on a spacecraft. In addition to comparing trajcc.tories on the basis of total transfer ΔV then, it is also useful to consider the payload mass that can be delivered using each trajectory type. This is probably a superior basis for comparison, since the throw weight available for mission instruments and equipment is the most critical element in the design of most missions. In addition, the total ΔV sum is misleading in the sense that most of it must be executed by a launch vehicle with a propulsion system and stage efficiency very different than the. spacecraft. Unfortunately, the relationship between ΔV and payload capability is not a simple. oat. A launch system and spacecraft must be completel y designed before the payload capability is firmly established. Nevertheless, there are simple design rules that may be applied to provide estimates of payload mass that could be delivered base.d on different transfers.

As one example, the payload capacity available reboard a small spacecraft in the 1000 kg range using different transfers is examined. The following comparisons are based on the use of a three-stage Delta II 6925 launch vehicle capable of injecting a wet spacecraft mass in the range of 1000 to 1100 kg onto any of the trajectories considered. The precise mass allowed is related to the required injection velocity by the performance results published by McDonnell Douglas.[9] These results contain considerable margin, so no additional margin is carried in the spacecraft designs considered here. Total propellant mass required is derived in each case. from the spacecraft ΔV requirement and the classical rocket equation, from which payload capacity can then be computed. Spacecraft-to-launch vehicle adapter mass is assumed to be 3% of the wet spacecraft mass. Total spacecraft structure mass is taken as 18%. The inert spacecraft propulsion system mass is a combination of 25 kg plus 20% of the total propellant mass carried, based on the assumption of using a small bi-propellant system with a thrust of approximately 400 Newtons (sized for a single large lunar orbit insertion burn) and a set of small reaction control engines. The average specific impulse of the system is 290 sec. Unusable propellant residuals equal to 0.5% are assumed.

Table 3 shows payload capabilities derived according to these assumptions for four different trajectory designs. The first is a theoretical Hohmann transfer. The Belbruno/Miller trajectory is also presented along with Sweetser's theoretical minimum ΔV case and the trajectory presented by the authors in this paper. This last

Table 2. Summary of Earth-to-Moon Transfers

TYPE	M()])111 ,	TOTAL ΔV (km/sec)	EARTH INJECTION ΔV (km/sec)	MOON INSERTION AV (kill/SCC)
Minimum[1]	3-Body	3.721	3.099	0.627
This Study	3-Body	3.824	3.194*	0.629*
Belbruno/Miller	Real World	3.838	3.187*	0.651
Biparabolic	2-Body	3.946	3.232	0.714
Hohmann	2-Body	3.959	3.140	0.819

*Includes mid-course maneuvers

trajectory appears twice in the table, with the second appearance representing the results of using a low-thrust electrothermally augmented thruster such as an arcjet (ypc with an average specific impulse of 510 sec. Such a system would have. demanding power requirements and a thrust level around ().2 Newtons. Nevertheless, it may be feasible for the type of low-energy transfer described in (his paper provided that the final lunar orbit insertion buro is executed in several stages, each AV under 100 m/s.

Low-thrust, high-cfficicocy systems arc certainly not an option for near-Hohmann cases in which a single fairly large impulse is required for lunar capture. For the low-energy cases io the example here, a large impulse is not required for capture, and the resulting efficiency advantage provides 161 kg (or 30%) Of usable payload more than a comparable Hohmann mission.

As compared to the Belbrooo-Miller trajectory, which extends to near the L_1 point of the Sun-Earth system (-1 .5x10^6 km from Earth), the trajectory in this study never leaves the Earth-Moon system. By remaining near the Earth throughout the trajectory, communications and operational requirements could be simplified. 10 addition, the Belbruno-Miller trajectory may have more restrictive launch windows than the trajectory presented iu this paper.

Table 3. Summary of Payload Mass Deliverable 10 the Moon

Trajectory Type	Maximum Payload Mass (Kg)
Hohmann	521.2
Belbruno/Miller[5-7]	556.8
Minimum[1]	615.8
This Study	570.3
This Study*	682.0

*Using a low-thrust electric thruster

Concluding Remarks and Future Work

While the trajectory found io this study has a low total AV compared to other designs, its uscs arc limited due to the long time-of-flight. Future work could examine a number Of aspects of this trajectory. The Earth-to-L_1 leg AV is still significantly higher than the theoretical minimum, and it is possible that a trajectory with a lower AV sad/or shorter flight time might exist. Also, the trajectory in this study was computed assuming circular lunar motion with no solar or other perturbing effects. A more realistic model could be used to examine whether the trajectory and its corresponding Av change significantly, and, if the solar perturbation could be used beneficially to reduce the AV incurred sad/or the time.-of-fliglll.

Another important aspect to consider is the navigation requirements for spacecraft on these types of trajectories. Since these trajectories will likely have high sensitivities to maneuver execution errors, the requirements to maintain navigational control of the spacecraft should be examined.

A final point to note is the possible application of the L_1-to-Moon leg for use as a transfer trajectory for future lunar operations when using L_1 as a staging point between the Earth and Moon. As computed io this study, the AV for this leg is near the theoretical minimum AV and has relatively small maneuver magnitudes at the L_1 departure and the lunar insertion locations (0.1 m/s and 62.7 m/s, respectively). This type of transfer from L_1 to the Moon would also have the advantage that there would be no restrictions on the launch window departing 1.1. Again, however, the flight time is long (--50 days), and future efforts could examine reducing this time while attempting to retain the low AV cost.

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